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Algebraic Compression of Quantum Circuits

arXiv:2104.00728
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arXiv:2108.03283 (to appear in SIMAX)

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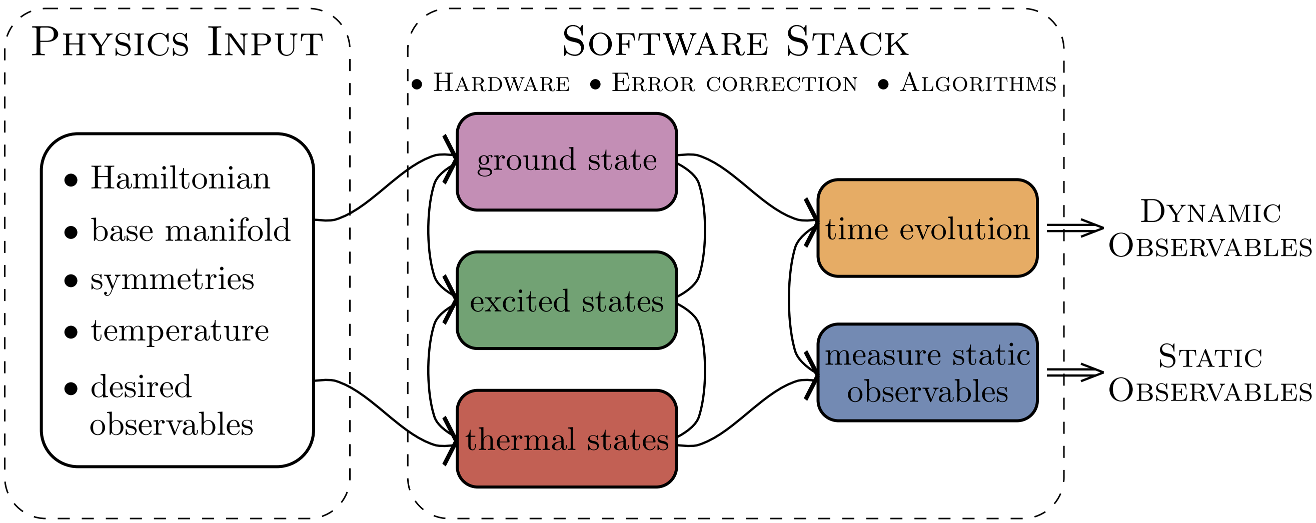
NC State IBM Q Hub Symposium
06/23/2022

With:
Efekan Kökcü, Thomas Steckmann (NCSU)
Eugene Dumitrescu & Yan Wang (ORNL)
Daan Camps, Roel van Beeumen, Lindsay Bassman, Bert de Jong (LBNL)
Jim Freericks (Georgetown)

Quantum Matter meets Quantum Computing



- Measuring correlation functions
- Preparing/measuring topological states
- Modeling driven/dissipative systems
- Time evolution via Lie algebraic decomposition and compression
- Thermodynamics

The problem of time evolution

$$|\psi\rangle \to |\psi(t)\rangle = U(t)|\psi\rangle$$

- Physics/chemistry simulation
- QAOA
- State preparation

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The problem of time evolution

$$|\psi\rangle \rightarrow |\psi(t)\rangle = U(t)|\psi\rangle$$

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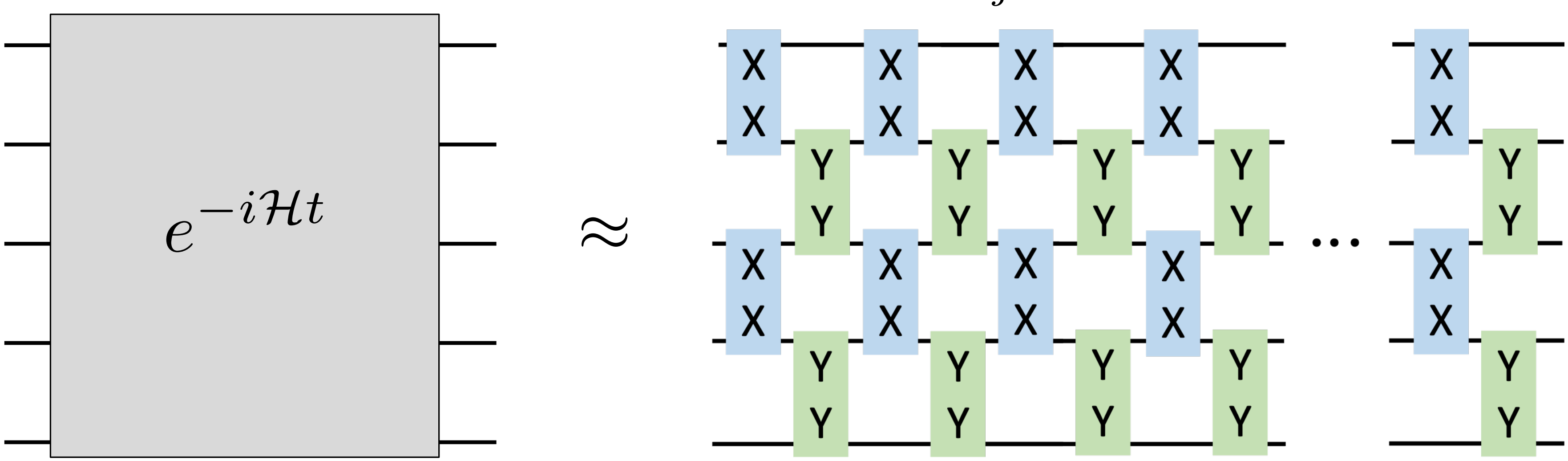
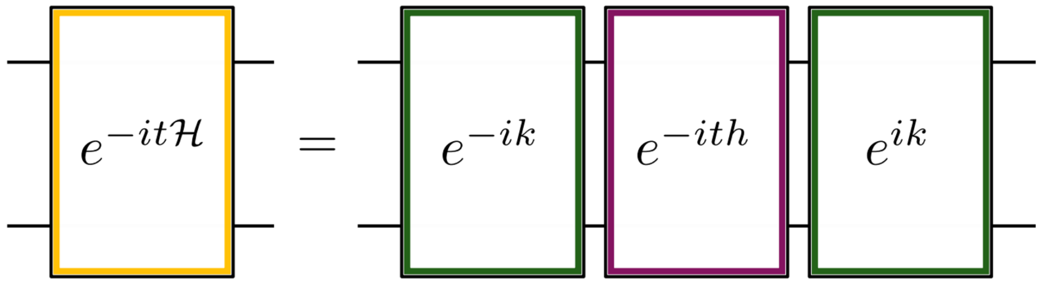
- QAOA
- State preparation

$$U(t) = e^{-i\mathcal{H}t} \quad , \mathcal{H} = \sum_j h_j \sigma^j$$



Alternate Approach #1: Cartan Decomposition



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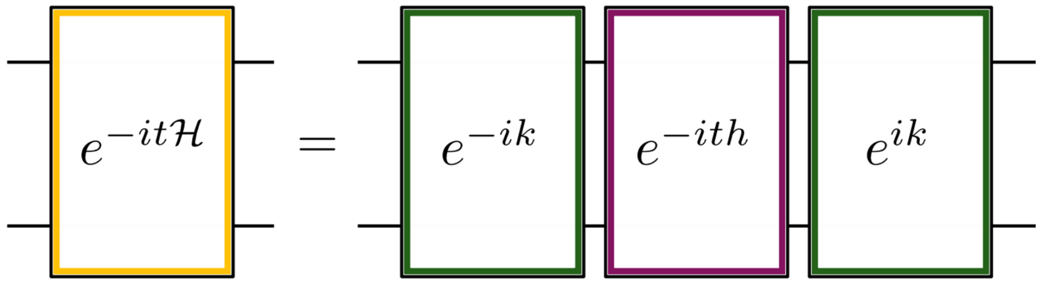
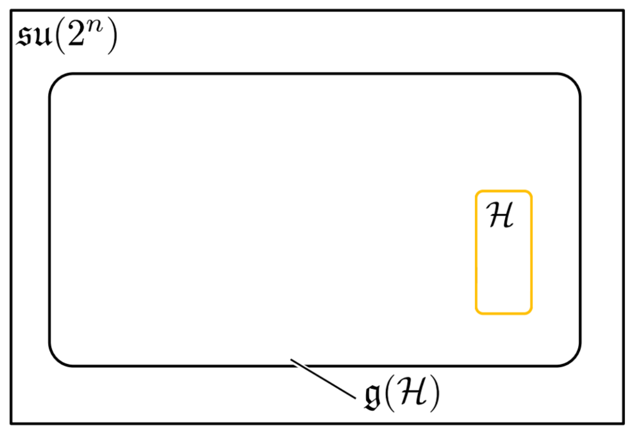


1. Form *Hamiltonian algebra* $\mathfrak{g}(\mathcal{H})$

$$\mathcal{H} = \sum_j h_j \sigma^j$$



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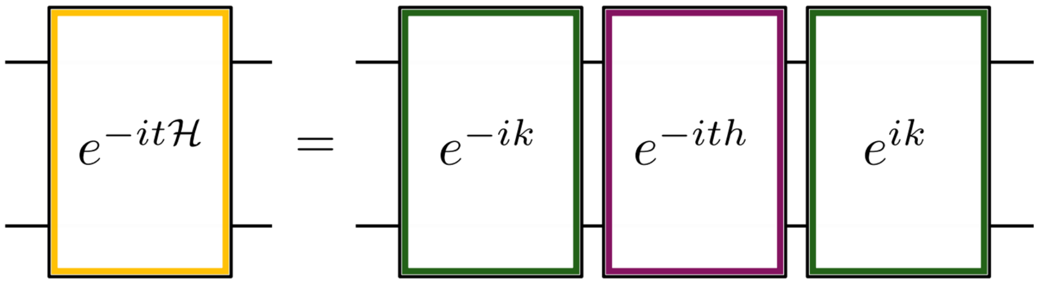

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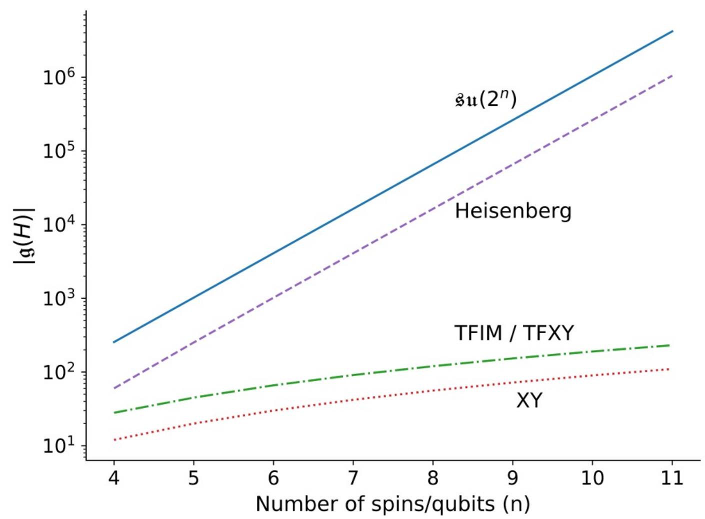

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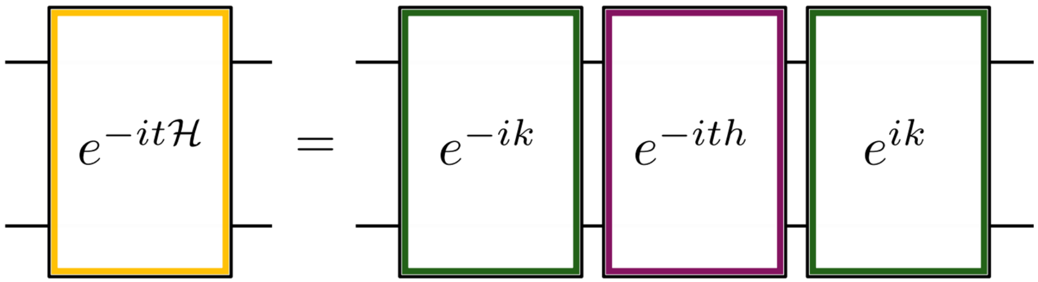
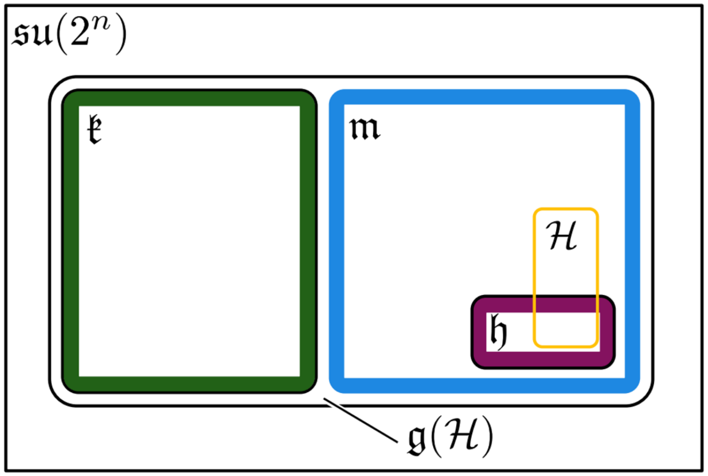


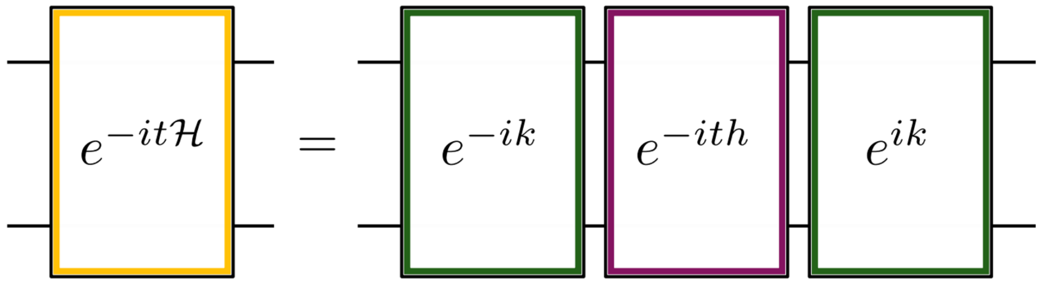
1. Form *Hamiltonian algebra* $\mathfrak{g}(\mathcal{H})$

2. Split the algebra via Cartan Decomposition

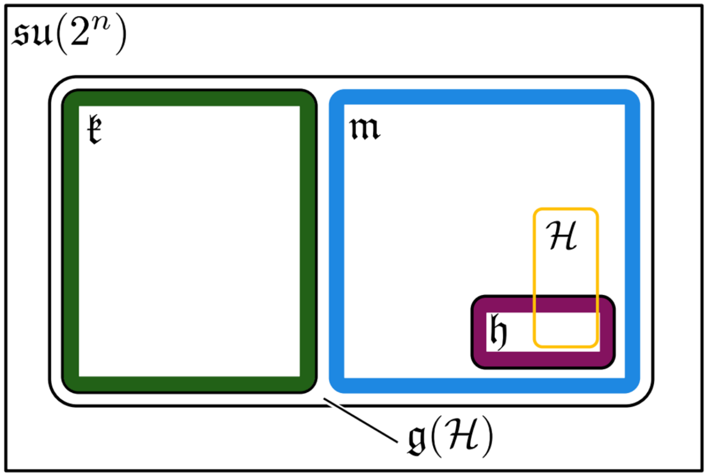
$$\mathcal{H} = KhK^{\dagger}$$



Alternate Approach #1: Cartan Decomposition



1. Form *Hamiltonian algebra* $\mathfrak{g}(\mathcal{H})$
2. Split the algebra via Cartan Decomposition
$$\mathcal{H} = KhK^\dagger$$
3. Optimize over parameters in K
4. Construct circuit for *K*
$$e^{-i\mathcal{H}t} = Ke^{-iht}K^\dagger$$



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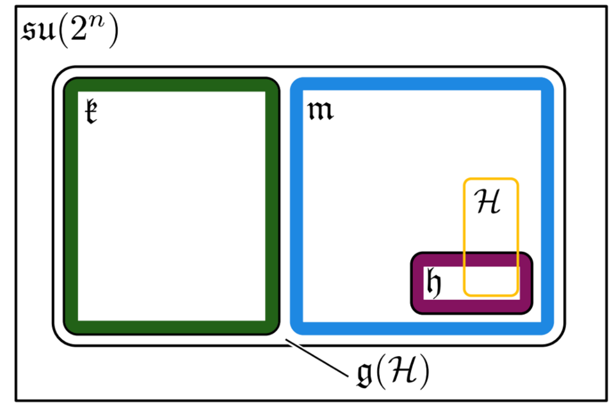
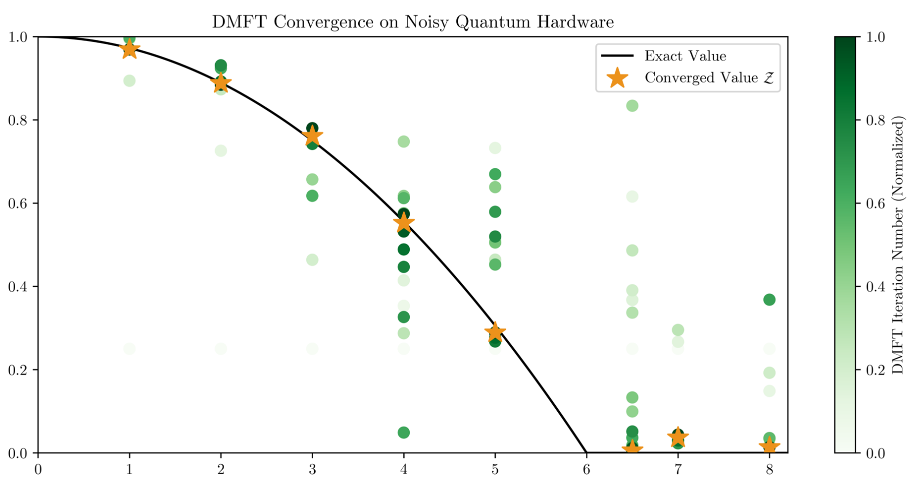
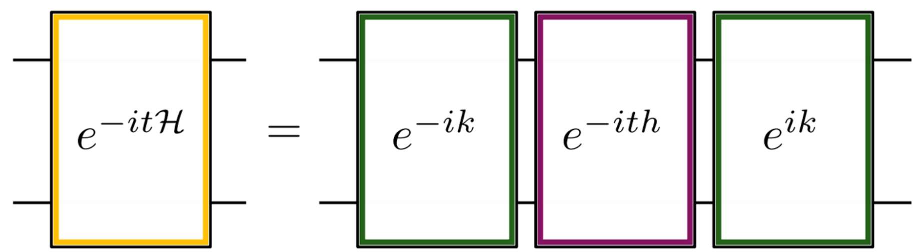
Alternate Approach #1: Cartan Decomposition

- $O(n^2)$ circuit for TFIM, TFXY, XY
- Applicable for any model
- Optimize only once for any time t
- Used this to obtain 1st ever DMFT phase diagram on IBM QC.







Alternate Approach #2: Algebraic Circuit Compression

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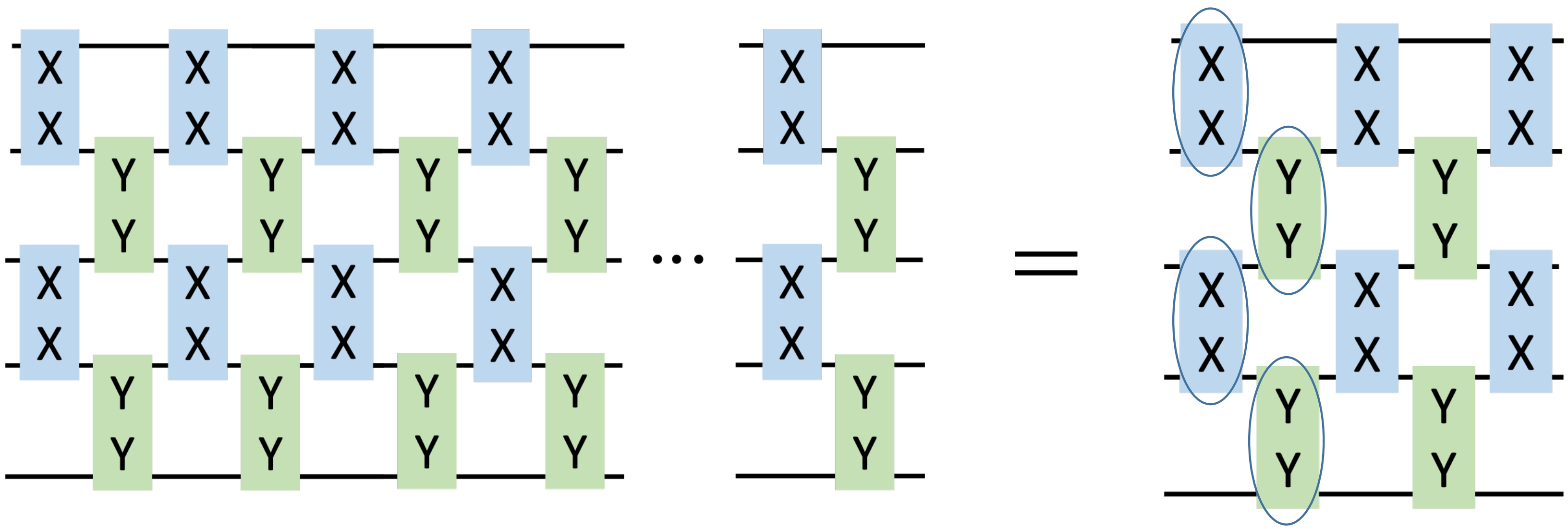
Alternate Approach #2: Algebraic Circuit Compression

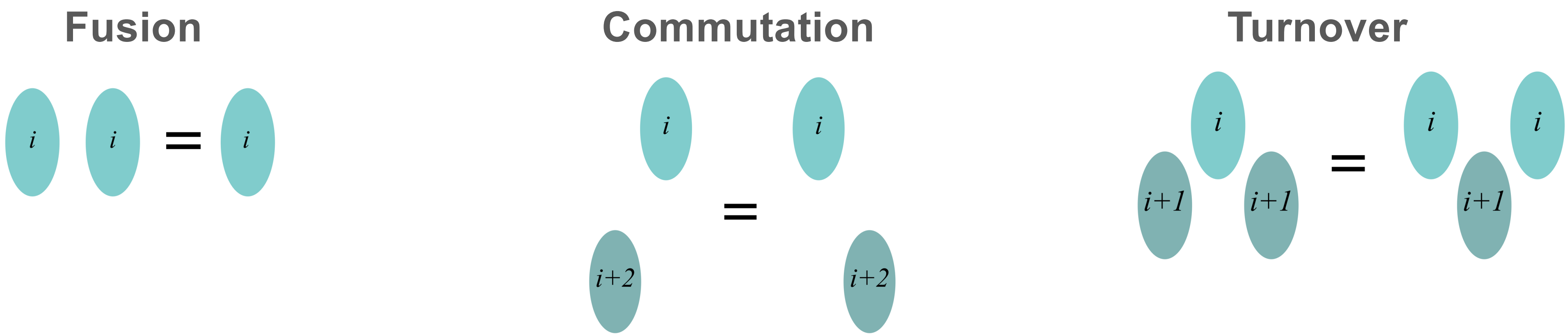
- We propose a **constructive,** Lie algebra based method which leads to fixed depth circuits for several models

- The method is **scalable** due to its “constructive” and “local” nature.



Block

We define an abstract object called "block" which satisfies:



Blocks will be mapped to certain quantum gates in a model specific way.

These properties are **local**! One needs to check only the neighbor gates to apply them, not the whole circuit.

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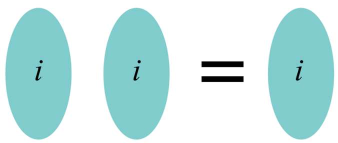
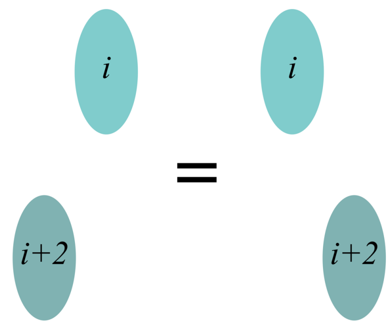
Alternate Approach #2: Algebraic Circuit Compression

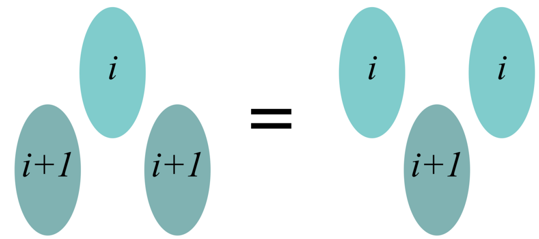
Fusion

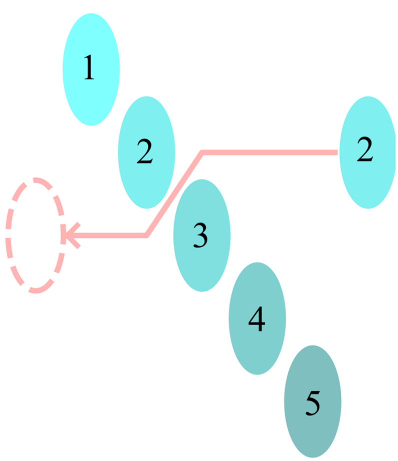


Commutation



Turnover





Alternate Approach #2: Algebraic Circuit Compression

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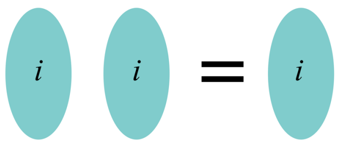

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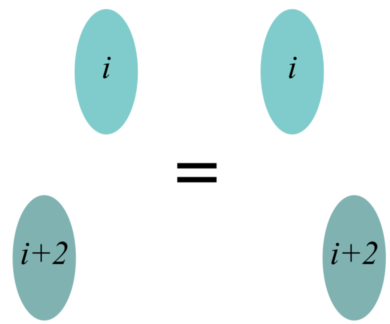

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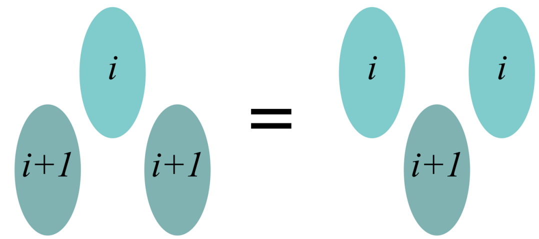

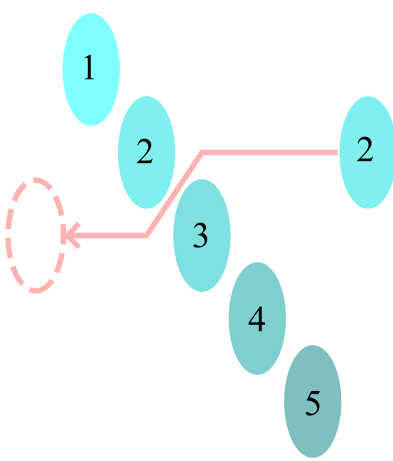

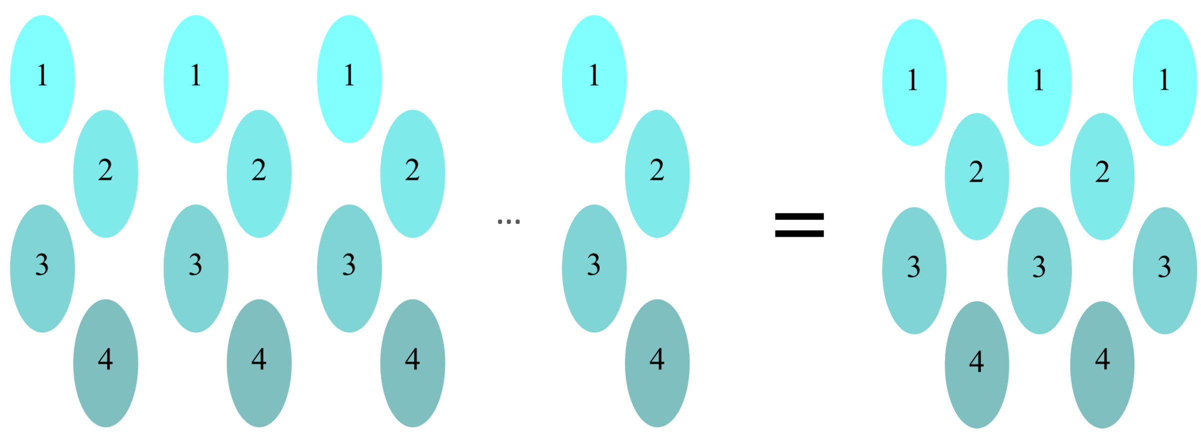

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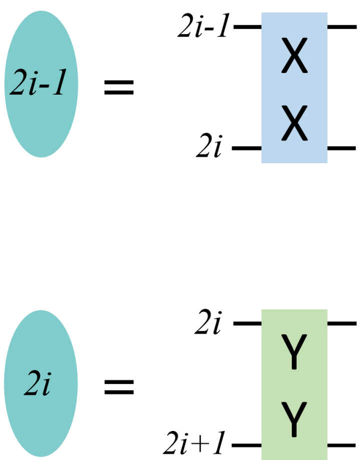
Alternate Approach #2: Algebraic Circuit Compression

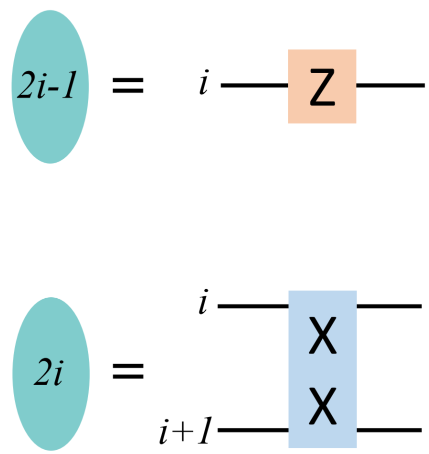
Kitaev Chain



n(n-1) CNOTs

n(n-1)/2 XX gates

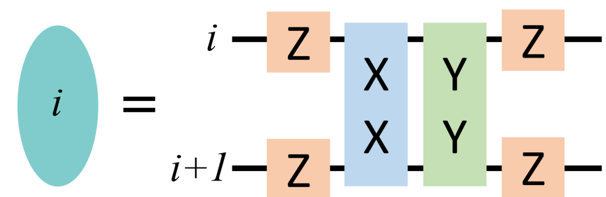
TFIM



2n(n-1) CNOTs

n(n-1) XX gates

TFXY



n(n-1) CNOTs

n(n-1) XX gates

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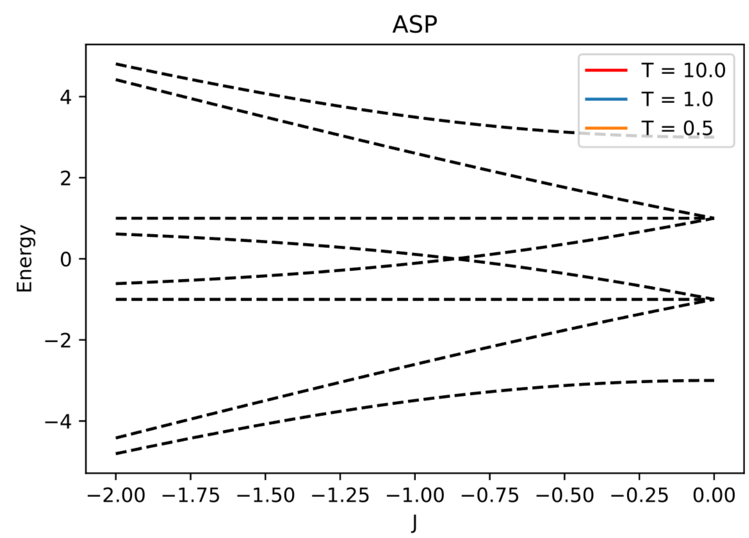
Alternate Approach #2: Algebraic Circuit Compression

Ising model

$$H = -2(X_1X_2 + X_2X_3) - (Z_1 + Z_2 + Z_3)$$



Ising model with J=0

$$H = -(Z_1 + Z_2 + Z_3)$$

$$|\psi\rangle = |000\rangle$$

$$H = J(t)(X_1X_2 + X_2X_3) - (Z_1 + Z_2 + Z_3)$$

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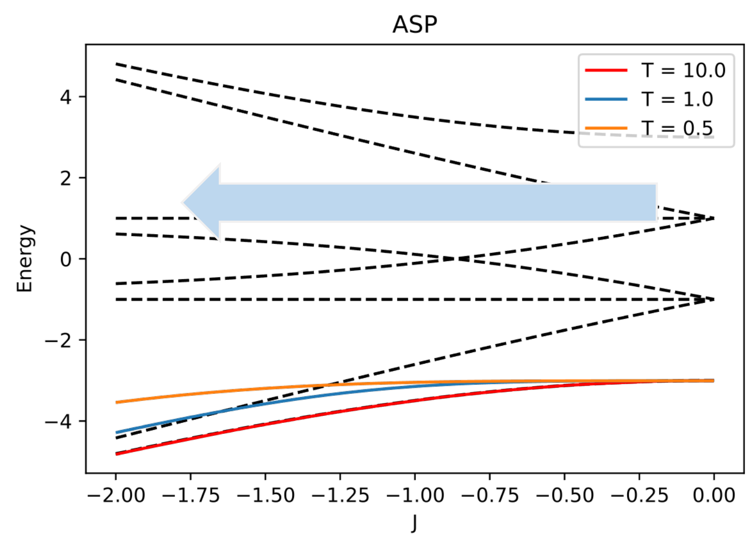

Ising model with J=0

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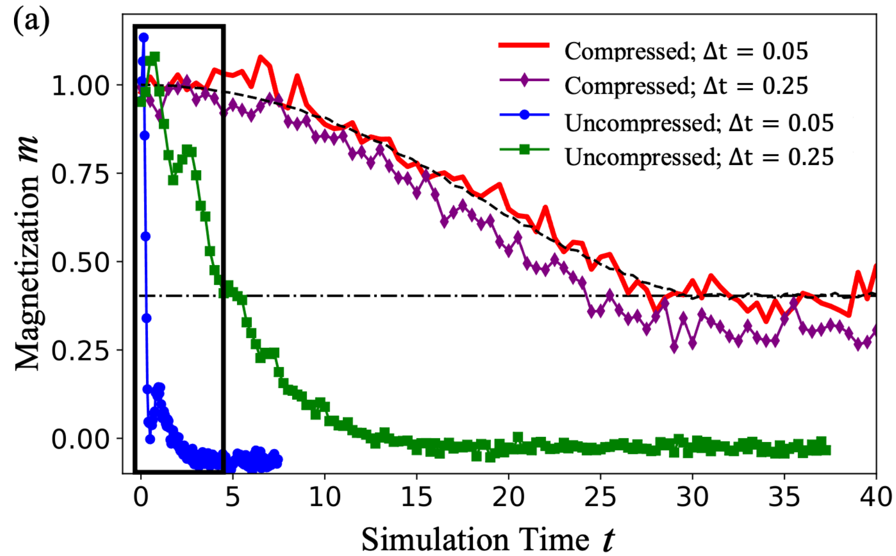
Alternate Approach #2: Algebraic Circuit Compression

$$\mathcal{H}_{ASP}(t) = J(t)\sum_{i=1}^{n-1} X_i X_{i+1} + h_z \sum_{i=1}^{n} Z_i \qquad \langle m(t) \rangle \equiv \frac{1}{n}\sum_i \sigma_i^z(t)$$



- Compressed circuits have 20 CNOT gates in total whereas Trotter circuits have increasing number of CNOTs as simulation time increases

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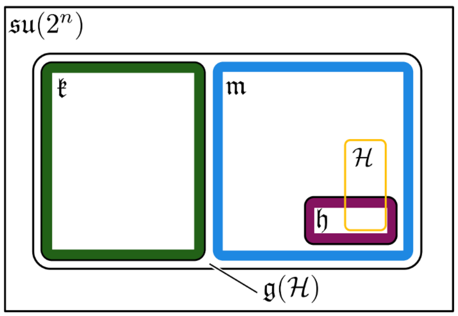
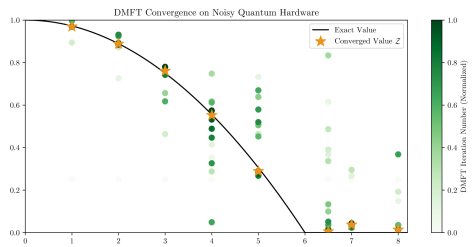
Conclusion

Cartan Decomposition

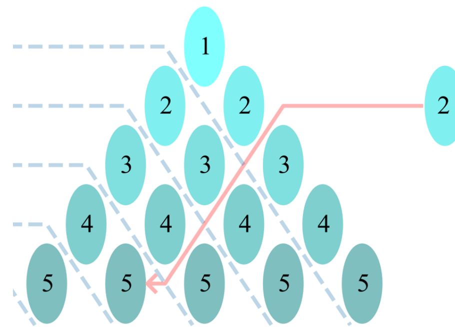


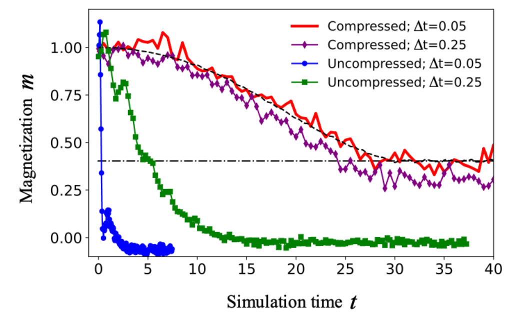


- Produces exact, fixed depth time evolution unitaries for any model.
- Constructive approach to base variational methods on.
- We have code available! https://github.com/kemperlab/cartan-quantum-synthesizer

arXiv:2104.00728, arXiv:2112.05688

Algebraic Compression





- We have a method to compress Trotter circuits down to a fixed depth circuit for 1-D nearest neighbor TFXY, TFIM, XY and Kitaev models.
- Based on 3 easy to check, local properties. Transpiler software
- We have code available! Check F3C, F3C++ and F3Cpy at https://github.com/QuantumComputingLab

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